Algebraic Expressions and Identities

Class 8 Mathematics

Complete Study Notes

Learning Objectives

Understand the structure of algebraic expressions and identify variables, constants, coefficients, and terms. Learn to perform addition, subtraction, and multiplication of algebraic expressions, simplify polynomial expressions, and apply standard algebraic identities to solve mathematical problems quickly and accurately.

Introduction to Algebraic Expressions and Identities

Algebra is one of the most important branches of mathematics because it allows us to represent numbers, relationships, and real-life situations using symbols and variables. Instead of working only with known numbers, algebra uses letters such as x, y, and z to represent unknown quantities. These letters help us solve equations, recognize patterns, and build mathematical models used in science, engineering, economics, and technology.

An algebraic expression is formed by combining variables, constants, and mathematical operations. Understanding how to simplify and manipulate these expressions is essential before learning equations, factorization, and higher algebra. This chapter also introduces algebraic identities, which are useful formulas that simplify lengthy calculations and improve problem-solving speed.

Basic Terms in Algebraic Expressions

An algebraic expression consists of different parts that work together to represent a mathematical quantity. A **variable** is a symbol, usually a letter, whose value may change depending on the situation. Common variables include x, y, and z. A **constant** is a fixed numerical value that never changes, such as 5, −3, or 12.

Every algebraic expression is made up of one or more **terms**. A term is formed by multiplying constants and variables together. The numerical part of a term is called its **coefficient**. For example, in the term $7x^2$, the coefficient is 7, while x^2 is the variable part.

Algebraic expressions are classified according to the number of terms they contain:

- An expression with one term is called a **monomial** (e.g., $4x^2$).
- An expression containing two terms is called a **binomial** (e.g., $3x + 5$).
- An expression containing three terms is called a **trinomial** (e.g., $x^2 + 2x + 1$).
- Expressions containing more than three terms are simply called **polynomials**.

Understanding these basic terms helps students identify the structure of expressions and perform algebraic operations correctly.

Addition and Subtraction of Algebraic Expressions

Addition and subtraction of algebraic expressions follow a simple but important rule: **only like terms can be added or subtracted**. Like terms have exactly the same variables raised to the same powers. Their numerical coefficients may differ, but the variable parts must be identical.

For example,
$2x^2 + 5x^2 = 7x^2$ (because both terms contain x^2).
Similarly,
$8xy - 3xy = 5xy$.

However, $4x$ and $4y$ are unlike terms because their variables are different. Likewise, $3x$ and $3x^2$ are unlike because the powers of the variables are different.

When adding or subtracting complete expressions, students first remove brackets wherever necessary and then combine all like terms carefully.
For example, $(2x^2 + 3x) + (5x^2 - x)$ becomes $7x^2 + 2x$.
Simplifying expressions by combining like terms is one of the most fundamental skills in algebra and is used throughout higher mathematics.

Multiplication of Algebraic Expressions

Multiplication of algebraic expressions begins with multiplying monomials. The numerical coefficients are multiplied together, while variables having the same base are combined using the laws of exponents.
For example: $(3x)(-5xy) = -15x^2y$.

When multiplying a monomial by a polynomial, the Distributive Property is applied. Every term inside the bracket must be multiplied by the outside term.
For example: $2x(3x + y)$ becomes $6x^2 + 2xy$.
The distributive property makes multiplication easier and forms the basis for multiplying larger algebraic expressions.

When multiplying two polynomials, each term of the first polynomial is multiplied by every term of the second polynomial. After multiplication, all like terms are combined to obtain the simplest possible expression.
For example: $(x + 4)(x + 2)$ becomes $x^2 + 4x + 2x + 8$, which simplifies to $x^2 + 6x + 8$.
Carefully organizing the multiplication process helps avoid mistakes and ensures accurate simplification.

Algebraic Identities

An algebraic identity is a mathematical statement that remains true for every possible value of its variables. Unlike ordinary equations, identities do not have specific solutions because they are always valid. Algebraic identities make calculations faster and help simplify lengthy expressions without repeated multiplication.

Standard Identities:

- $(a + b)^2 = a^2 + 2ab + b^2$
- $(a - b)^2 = a^2 - 2ab + b^2$
- $(a + b)(a - b) = a^2 - b^2$
- $(x + a)(x + b) = x^2 + (a + b)x + ab$

These identities are frequently used to expand expressions, factorize polynomials, verify equations, and perform mental calculations.
For example, 103^2 can be written as $(100 + 3)^2$. Applying the identity:
$100^2 + 2(100)(3) + 3^2 = 10000 + 600 + 9 = 10609$.
Using identities saves time and reduces the amount of manual multiplication required.

Applications of Algebraic Expressions and Identities

Algebraic expressions are used in almost every field involving mathematics. Engineers use them while designing structures, scientists use them to describe physical laws, economists use them to represent financial models, and computer programmers use algebraic formulas while developing algorithms. Algebraic identities help simplify calculations in geometry, trigonometry, coordinate geometry, and advanced algebra.

Practice Questions

1. Identify the coefficient and variable in $9x^2$.
2. Simplify $(4x + 3x - 2x)$.
3. Multiply $3x(2x + 5)$.
4. Multiply $(x + 4)(x + 2)$.
5. Expand $(a + b)^2$ using the standard identity.

Answer Key

- **ANSWER 1:** In $9x^2$, the coefficient is 9 and the variable part is x^2.
- **ANSWER 2:** $4x + 3x - 2x = 5x$.
- **ANSWER 3:** $3x(2x + 5) = 6x^2 + 15x$.
- **ANSWER 4:** $(x + 4)(x + 2) = x^2 + 6x + 8$.
- **ANSWER 5:** $(a + b)^2 = a^2 + 2ab + b^2$.