

Cubes and Cube Roots

Class 8 Mathematics

Complete Study Notes

Learning Objectives

Understand the concept of cube numbers and cube roots. Learn the properties of perfect cubes, identify cube numbers using mathematical patterns, apply prime factorization to determine perfect cubes, find cube roots using different methods, and calculate cube roots of large numbers and rational fractions with confidence.

Introduction to Cubes and Cube Roots

Cube numbers and cube roots are important mathematical concepts that build upon multiplication and exponents. A cube number is obtained by multiplying a number by itself three times, while a cube root is the number that, when multiplied by itself three times, produces the original number. These concepts are widely used in algebra, geometry, mensuration, engineering, and higher mathematics.

Understanding cubes and cube roots helps students solve numerical problems efficiently and strengthens logical reasoning. Recognizing the properties and patterns of perfect cubes also makes mental calculations much faster.

Understanding Cube Numbers

A cube number, also called a perfect cube, is obtained by multiplying a natural number by itself three times. For example, $4 \times 4 \times 4 = 64$, so 64 is a perfect cube. Similarly, $7 \times 7 \times 7 = 343$, making 343 another perfect cube.

Cubes from 1 to 10: $1^3 = 1$, $2^3 = 8$, $3^3 = 27$, $4^3 = 64$, $5^3 = 125$, $6^3 = 216$, $7^3 = 343$, $8^3 = 512$, $9^3 = 729$, and $10^3 = 1000$.

Properties: The cube of an even number is always even, while the cube of an odd number is always odd. A perfect cube can also end only with multiples of three zeros. Numbers ending with one or two trailing zeros cannot be perfect cubes.

Prime Factorization and Perfect Cubes

Prime factorization is one of the most reliable methods for identifying perfect cubes. Every perfect cube has its prime factors arranged in complete groups of three identical factors. If even one prime factor is left ungrouped, the number is not a perfect cube.

This property is also useful for solving practical questions. When asked to find the smallest number that should be multiplied by a given number to make it a perfect cube, students first perform prime factorization and identify the missing factors required to complete each group of three.

Similarly, if a question asks for the smallest number by which a given number should be divided to obtain a perfect cube, the extra prime factors that do not form complete triplets are removed. This simple technique makes solving perfect cube problems systematic and accurate.

Finding Cube Roots

A cube root is the inverse operation of finding a cube. If $6^3 = 216$, then $\sqrt[3]{216} = 6$. Cube roots are represented using the symbol $\sqrt[3]{}$ and are commonly used in algebra and higher mathematics.

The Prime Factorization Method is one of the easiest ways to find the cube root of a perfect cube. The given number is first expressed as the product of its prime factors. Identical prime factors are grouped into sets of three, and one factor is taken from each group. Multiplying these selected factors gives the cube root.

For example,

$$216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$$

Grouping them into triplets gives:

$$(2 \times 2 \times 2) \times (3 \times 3 \times 3)$$

Taking one factor from each group,

$$\sqrt[3]{216} = 2 \times 3 = 6.$$

This method provides an accurate way to calculate cube roots of large perfect cubes.

Cube Roots of Large Numbers

Large perfect cubes can also be solved using the Estimation Method, which is faster than complete prime factorization. The number is divided into groups of three digits starting from the right. The last digit helps determine the unit digit of the cube root, while the remaining group helps identify the first digit by comparing it with nearby perfect cubes.

For example, to find the cube root of 2744, the number is grouped as 2 | 744. Since $1^3 = 1$ and $2^3 = 8$, the first digit is 1 because 2 lies between 1 and 8. The last digit 4 indicates the unit digit is 4, giving the answer 14.

This estimation technique allows students to calculate cube roots mentally for many large perfect cubes.

Cube Roots of Rational Numbers

Cube roots can also be found for rational fractions. The numerator and denominator are treated separately by finding their individual cube roots.

For example,

$$\sqrt[3]{(64/125)} = \sqrt[3]{64} / \sqrt[3]{125} = 4/5.$$

This property makes calculations involving fractions much simpler and is frequently used in algebraic simplification.

Importance of Cubes and Cube Roots

Cube numbers and cube roots are widely used in geometry for calculating volumes of cubes and cuboids, in engineering and architecture for structural measurements, and in science and technology for solving mathematical models. These concepts also provide a strong foundation for algebra, coordinate geometry, and higher mathematics studied in later classes.

Practice Questions

1. Is 729 a perfect cube?
2. Find the cube of 12.
3. Find the cube root of 512.
4. Which smallest number should be multiplied by 72 to make it a perfect cube?
5. Find the cube root of $64/125$.

Answer Key

- **ANSWER 1:** Yes. $729 = 9^3$, so it is a perfect cube.
- **ANSWER 2:** $12^3 = 12 \times 12 \times 12 = 1728$.
- **ANSWER 3:** $\sqrt[3]{512} = 8$, because $8 \times 8 \times 8 = 512$.
- **ANSWER 4:** Prime factorization of $72 = 2^3 \times 3^2$. One more factor of 3 is required to complete the triplet. Therefore, the smallest number is 3.
- **ANSWER 5:** $\sqrt[3]{(64/125)} = \sqrt[3]{64} \div \sqrt[3]{125} = 4/5$.