Playing with Numbers

Class 8 Mathematics

Complete Study Notes

Learning Objectives

Understand how numbers can be represented using their place values through the generalized form of numbers. Learn interesting number patterns formed by reversing digits, understand the mathematical logic behind divisibility tests, solve cryptarithms using logical reasoning, and develop problem-solving skills through number puzzles.

Introduction

Numbers are not just symbols used for counting—they follow many interesting patterns and rules. In this chapter, students learn how numbers can be written in a generalized algebraic form using their digits and place values. This representation helps explain why many divisibility rules work and reveals fascinating relationships between numbers.

The chapter also introduces cryptarithms, where letters replace digits in mathematical calculations. These puzzles encourage logical thinking and improve reasoning skills while making mathematics enjoyable. Understanding these concepts strengthens problem-solving abilities and builds a strong foundation for higher mathematics.

Generalised Form of Numbers

Every number is made up of digits placed at different place values. Instead of writing a number directly, it can be expressed as the sum of its digits multiplied by their respective place values. This is called the generalised form of a number.

> **Two-digit number:** For a number *ab*, where *a* represents the tens digit and *b* represents the ones digit:
> **ab = 10a + b**
> **Three-digit number:** For a number *abc*:
> **abc = 100a + 10b + c**

Writing numbers in this form helps students understand the value contributed by each digit. It also forms the basis for proving divisibility rules and solving mathematical puzzles.

Number Reversals and Interesting Patterns

The generalized form can be used to study numbers when their digits are reversed. If the number *ab* is reversed, it becomes *ba = 10b + a*.

Adding these two numbers gives:
ab + ba = (10a + b) + (10b + a) = 11(a + b)
Since the result is always a multiple of 11, the sum of any two-digit number and its reverse is divisible by 11.

Similarly, subtracting them gives:
ab − ba = (10a + b) − (10b + a) = 9(a − b)
The difference is always a multiple of 9. These patterns show how algebra can explain many interesting properties of numbers instead of relying on observation alone.

Divisibility Tests

Divisibility tests help determine whether one number can be divided by another without performing complete division. These rules are based on the place values of digits and make calculations much faster.

- A number is divisible by **2** if its ones digit is 0, 2, 4, 6, or 8. Since only the last digit determines whether a number is even, the remaining digits do not affect divisibility by 2.
- A number is divisible by **5** if its ones digit is 0 or 5. Again, only the units digit matters because every multiple of 10 is already divisible by 5.
- For divisibility by **3 and 9**, the sum of all the digits is considered. If the sum of the digits is divisible by 3, then the number itself is divisible by 3. Likewise, if the digit sum is divisible by 9, then the entire number is divisible by 9.

Cryptarithms and Letter Puzzles

Cryptarithms are mathematical puzzles in which letters are used instead of digits. The objective is to determine which digit each letter represents while satisfying the given arithmetic operation.

Two important rules must always be remembered:
1. Every letter represents only one unique digit.
2. The first digit of any number cannot be zero because numbers do not begin with zero.

By solving one column at a time, students can determine the missing digits using logical reasoning instead of guessing randomly. For example, looking at a units column: A + 3 = 5, so A = 2.

Solving Multiplication Puzzles

Some cryptarithms involve multiplication instead of addition. In such questions, students carefully examine the multiplication process, carry values, and the units digit to determine the correct value of each letter.

Logical elimination plays an important role in solving these puzzles. Each possible digit is tested against the given conditions until the correct solution is found. These activities improve observation skills, logical thinking, and confidence in mathematical reasoning.

Importance of Playing with Numbers

The concepts in this chapter are useful beyond classroom mathematics. Divisibility tests simplify calculations and help students solve problems more efficiently. The generalized form of numbers

develops an understanding of algebraic thinking, while cryptarithms encourage logical reasoning and analytical skills. These ideas prepare students for advanced topics in algebra and number theory and make mathematics more interesting by showing that numbers follow beautiful and predictable patterns.

Practice Questions

1. Write the generalized form of the three-digit number abc.
2. Is 936 divisible by 9? Give a reason.
3. Explain why the sum of a two-digit number and its reverse is always divisible by 11.
4. What are the two important rules for solving cryptarithms?
5. Is 745 divisible by 5? Explain your answer.

Answer Key

- **ANSWER 1:** The generalized form is 100a + 10b + c, where a, b, and c represent the hundreds, tens, and ones digits respectively.
- **ANSWER 2:** Yes. The sum of the digits is 9 + 3 + 6 = 18, and 18 is divisible by 9, so 936 is divisible by 9.
- **ANSWER 3:** If ab = 10a + b and ba = 10b + a, then their sum is 11(a + b), which is always divisible by 11.
- **ANSWER 4:** Each letter represents only one unique digit, and the first digit of a number cannot be zero.
- **ANSWER 5:** Yes. The ones digit of 745 is 5, so it is divisible by 5.