Rational Numbers

Class 8 Mathematics

Complete Study Notes

Learning Objectives

Understand the concept of rational numbers and how they extend the number system beyond integers. Learn how to write rational numbers in standard form, represent them on a number line, compare and simplify fractions, identify additive and multiplicative inverses, apply the properties of rational numbers, and find rational numbers between any two given numbers through solved examples and practice questions.

Introduction to Rational Numbers

Rational numbers are an important part of mathematics because they help us represent quantities that cannot always be expressed as whole numbers. Everyday situations such as sharing food equally, measuring distances, calculating discounts, and expressing probabilities often require the use of fractions.

> **Definition:** Every rational number can be written in the form p/q, where p and q are integers and q is not equal to zero. Since division by zero is impossible in mathematics, a fraction with a denominator of zero is not considered a rational number.

Rational numbers include positive fractions, negative fractions, zero, whole numbers, and integers. For example, 5 can be written as 5/1, and 0 can be written as 0/1. This means that every integer and every whole number is also a rational number.

Standard Form of Rational Numbers

A rational number is said to be in its standard form when the numerator and denominator have no common factor other than 1, and the denominator is always positive. To write a rational number in standard form, divide both the numerator and denominator by their greatest common factor. If the denominator is negative, shift the negative sign to the numerator so that the denominator becomes positive.

For example, the fraction 12/18 is simplified to 2/3, while 3/-5 is written as -3/5 in standard form. Writing rational numbers in their simplest form makes calculations easier and helps compare different fractions accurately.

Absolute Value of Rational Numbers

The absolute value of a rational number represents its distance from zero on the number line without considering its direction. Therefore, both positive and negative forms of the same rational number have the same absolute value.

For example, the absolute value of 3/5 and -3/5 is 3/5. Absolute value is always non-negative because distance can never be negative.

Properties of Addition

Addition of rational numbers follows several important mathematical properties. The **Closure Property** states that the sum of any two rational numbers is always another rational number. The **Commutative Property** shows that changing the order of addition does not change the answer. Similarly, the **Associative Property** explains that changing the grouping of three or more rational numbers does not affect the final sum.

The number 0 is called the **Additive Identity** because adding zero to any rational number leaves it unchanged. Every rational number also has an **Additive Inverse**, which is the same number with the opposite sign. Adding a rational number and its additive inverse always gives zero.

Properties of Multiplication

Multiplication of rational numbers also follows several important properties. The product of two rational numbers is always another rational number, satisfying the Closure Property. Multiplication is both Commutative and Associative, meaning the order and grouping of numbers do not change the final product.

The number 1 is called the **Multiplicative Identity** because multiplying any rational number by one leaves it unchanged. Every non-zero rational number also has a **Multiplicative Inverse**, or reciprocal, obtained by interchanging the numerator and denominator. Multiplying a rational number by its reciprocal always gives one.

Multiplication also satisfies the **Distributive Property**, where multiplication can be distributed over addition and subtraction to simplify calculations.

Subtraction and Division of Rational Numbers

Although subtraction and division of rational numbers are always possible (except division by zero), they do not satisfy all mathematical properties. Both operations satisfy the Closure Property, but neither follows the Commutative Property nor the Associative Property.

This means changing the order or grouping of numbers during subtraction or division changes the final answer. Students should therefore solve these operations carefully by following the correct order of calculation.

Division by a rational number is performed by multiplying the first rational number with the reciprocal of the second rational number. However, division by zero is undefined and should never be attempted.

Representation on the Number Line

Rational numbers can be represented accurately on a number line by dividing the space between two consecutive integers into equal parts according to the denominator. The numerator then determines the exact position of the rational number.

Positive rational numbers are located to the right of zero, while negative rational numbers lie to the left. The number line provides an easy visual method for comparing rational numbers and understanding their relative positions.

Finding Rational Numbers Between Two Numbers

One of the unique properties of rational numbers is that infinitely many rational numbers exist between any two given rational numbers. One simple method is to calculate the average of the two numbers, while another method involves converting both fractions into equivalent fractions with a common denominator before identifying numbers between them.

This property makes the rational number system dense, meaning there is never a situation where two rational numbers have no other rational number between them.

Applications of Rational Numbers

Rational numbers are used extensively in everyday life. They help us calculate profits and discounts while shopping, measure lengths and weights, prepare recipes using fractional quantities, calculate marks and percentages, divide objects equally, and solve scientific and engineering problems involving measurements and ratios.

Mastering rational numbers provides the foundation for algebra, geometry, and many advanced mathematical concepts taught in higher classes.

Practice Questions

1. Write 18/24 in standard form.
2. Find the additive inverse and multiplicative inverse of 5/8.
3. Represent −3/4 on the number line.
4. Find one rational number between 2/5 and 3/5.
5. Does subtraction satisfy the commutative property for rational numbers? Explain.

Answer Key

- **ANSWER 1:** The greatest common factor of 18 and 24 is 6. Dividing both numerator and denominator by 6 gives 3/4, which is the standard form.
- **ANSWER 2:** The additive inverse of 5/8 is −5/8, while its multiplicative inverse is 8/5.
- **ANSWER 3:** Divide the interval between 0 and −1 into four equal parts. The third division from zero represents −3/4.
- **ANSWER 4:** One rational number between 2/5 and 3/5 is 1/2.
- **ANSWER 5:** No. Subtraction is not commutative because changing the order of subtraction changes the answer. For example, 2/3 − 1/3 = 1/3, whereas 1/3 − 2/3 = −1/3.