

Squares and Square Roots

Class 8 Mathematics

Complete Study Notes

Learning Objectives

Understand the concept of square numbers and square roots. Learn the properties and patterns of perfect squares, identify square numbers quickly, generate Pythagorean triplets, calculate the square of numbers ending in 5, and find square roots using repeated subtraction and prime factorization methods.

Introduction to Squares and Square Roots

Squares and square roots are important concepts in mathematics that appear in algebra, geometry, mensuration, and higher-level calculations. A square number is obtained by multiplying a number by itself, while a square root is the number that, when multiplied by itself, produces the original number. Understanding these concepts helps students solve mathematical problems more efficiently and builds a strong foundation for advanced topics.

Perfect squares follow several interesting patterns and mathematical properties that make calculations faster and easier. Learning these patterns also improves mental mathematics and problem-solving skills.

Understanding Square Numbers

A square number, also known as a perfect square, is obtained when a natural number is multiplied by itself. For example, $(5 \times 5 = 25)$, so 25 is a perfect square. Similarly, $(12 \times 12 = 144)$, making 144 another perfect square.

Properties of Perfect Squares: A square number can only end with the digits 0, 1, 4, 5, 6, or 9. Numbers ending in 2, 3, 7, or 8 can never be perfect squares. Similarly, a perfect square can only end with an even number of zeros. If a number ends with an odd number of zeros, it cannot be a perfect square.

Another important property is that the square of an even number is always even, while the square of an odd number is always odd. These simple observations help students recognize perfect squares without performing lengthy calculations.

Interesting Patterns in Square Numbers

Square numbers follow several fascinating mathematical patterns. One of the most important is that the sum of the first n consecutive odd numbers always equals n^2 . For example,

$$1 + 3 + 5 + 7 + 9 = 25 = 5^2$$

Another useful pattern is that between the squares of two consecutive natural numbers, there are exactly $2n$ non-square numbers. For example, between $3^2 = 9$ and $4^2 = 16$, there are six numbers (10, 11, 12, 13, 14, and 15).

Students also learn that adding two consecutive triangular numbers always produces a perfect square. These mathematical relationships make it easier to identify patterns and understand how numbers are connected.

Pythagorean Triplets and Squares Ending in 5

A Pythagorean Triplet consists of three natural numbers that satisfy the equation:

$$a^2 + b^2 = c^2$$

Examples include (3, 4, 5) and (5, 12, 13). A general formula can also be used to generate many Pythagorean triplets:

$(2m, m^2 - 1, m^2 + 1)$ where $m > 1$.

Students also learn a quick shortcut for finding the square of numbers ending in 5. Multiply the digit before 5 by its next consecutive number and write 25 at the end.

For example:

$$25^2 = 625$$

$$35^2 = 1225$$

$$75^2 = 5625$$

This simple trick allows students to calculate squares mentally without long multiplication.

Finding Square Roots

A square root is the inverse operation of squaring a number. If $12^2 = 144$, then the square root of 144 is 12. Square roots are represented using the radical symbol $\sqrt{}$ and are widely used in mathematics and science.

There are several methods for finding square roots. In the **Repeated Subtraction Method**, consecutive odd numbers are subtracted from the given number until zero is reached. The total number of successful subtractions gives the square root.

Another widely used method is **Prime Factorization**. The given number is first expressed as the product of its prime factors. Identical factors are then grouped into pairs, and one factor is taken from each pair. Multiplying these selected factors gives the square root of the original number.

Both methods help students understand the mathematical meaning of square roots rather than simply memorizing answers.

Importance of Squares and Square Roots

Squares and square roots are used in geometry, construction, architecture, engineering, computer science, physics, and statistics. They help calculate areas, solve equations, determine distances, and understand many real-life mathematical situations. Mastering these concepts also prepares students for algebra, coordinate geometry, trigonometry, and higher mathematics in future classes.

Practice Questions

1. Is 441 a perfect square?
2. Find the square of 85 using the shortcut method.
3. Find the square root of 144 using prime factorization.
4. Write one Pythagorean triplet.
5. How many non-square numbers lie between 8^2 and 9^2 ?

Answer Key

- **ANSWER 1:** Yes. $441 = 21 \times 21$, so it is a perfect square.
- **ANSWER 2:** $85^2 = 7225$. Multiply 8 by 9 to get 72 and write 25 at the end.
- **ANSWER 3:** $144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$. Pair the identical factors to get $2 \times 2 \times 3 = 12$. Therefore, $\sqrt{144} = 12$.
- **ANSWER 4:** One Pythagorean triplet is (3, 4, 5) because $3^2 + 4^2 = 5^2$.
- **ANSWER 5:** Between $8^2 = 64$ and $9^2 = 81$, there are 16 non-square numbers, following the rule $2 \times 8 = 16$.