

Playing with Numbers

Class 6 Mathematics

Complete Study Notes

Learning Objectives

In this chapter, students will develop a strong understanding of numbers and their properties. They will learn how to identify factors and multiples, distinguish between prime and composite numbers, recognize even and odd numbers, understand co-prime and twin prime numbers, use the Sieve of Eratosthenes to find prime numbers, perform prime factorization, and calculate the Highest Common Factor (HCF). These concepts build a strong foundation for advanced mathematics and improve logical thinking and problem-solving skills.

Introduction to Playing with Numbers

The chapter Playing with Numbers introduces students to the fascinating world of number patterns and relationships. Every number has unique properties that help us solve mathematical problems more efficiently. Understanding factors, multiples, prime numbers, divisibility, and HCF makes calculations easier and prepares students for higher-level mathematics.

These concepts are widely used in everyday life, from arranging objects into equal groups to solving practical problems involving sharing, measurement, and scheduling.

Factors and Multiples

A factor of a number is a number that divides it exactly without leaving any remainder. Every number has at least two factors: 1 and the number itself. For example, the factors of 24 are 1, 2, 3, 4, 6, 8, 12, and 24.

A multiple is obtained by multiplying a number by whole numbers. Multiples continue endlessly. For example, the multiples of 6 are 6, 12, 18, 24, 30, 36, and so on.

Understanding the relationship between factors and multiples helps students solve questions involving divisibility, common factors, and mathematical patterns.

Prime, Composite, Even and Odd Numbers

Numbers can be classified based on their factors.

- A **prime number** has exactly two factors—1 and the number itself. Examples include 2, 3, 5, 7, 11, and 13. The number 2 is the only even prime number.
- A **composite number** has more than two factors. Numbers such as 4, 6, 8, 9, and 12 are composite numbers because they can be divided by numbers other than 1 and themselves.
- An **even number** is completely divisible by 2, whereas an **odd number** is not divisible by 2.

Important Note: The number 1 is a special number because it is neither prime nor composite.

Co-prime Numbers and Twin Primes

Two numbers are called **co-prime numbers** if they have only one common factor, which is 1. For example, 8 and 15 are co-prime because they do not share any common factor other than 1.

Twin prime numbers are pairs of prime numbers whose difference is exactly 2. Examples include (3, 5), (5, 7), (11, 13), and (17, 19). Although the numbers are close together, both remain prime.

Learning these special types of numbers helps students understand number relationships and prepares them for more advanced mathematical concepts.

Sieve of Eratosthenes

The Sieve of Eratosthenes is a simple and systematic method for finding all prime numbers up to a given limit. Students begin by writing numbers in order, usually from 1 to 100. The number 1 is removed because it is neither prime nor composite.

Starting with 2, every multiple of 2 is crossed out except 2 itself. The same process is repeated for 3, 5, 7, and the remaining uncrossed numbers. The numbers left uncrossed at the end are all prime numbers.

This method provides an easy way to identify prime numbers quickly without checking every number individually.

Prime Factorization

Prime factorization means expressing a number as the product of its prime factors. Every composite number can be written uniquely as a multiplication of prime numbers.

For example,

- $24 = 2 \times 2 \times 2 \times 3$
- $60 = 2 \times 2 \times 3 \times 5$

Prime factorization makes it easier to simplify mathematical calculations and plays an important role in finding the Highest Common Factor (HCF).

Highest Common Factor (HCF)

The Highest Common Factor (HCF) is the greatest number that divides two or more numbers exactly. Finding the HCF is useful in simplifying fractions, arranging objects into equal groups, and solving real-life mathematical problems.

Students can calculate the HCF using three common methods. The **factor method** involves listing all the factors of each number and selecting the greatest common factor. The **prime factorization method** finds the common prime factors and multiplies the lowest powers of the common factors.

The **division method** repeatedly divides the larger number by the smaller one until the remainder becomes zero, making it the fastest method for larger numbers.

Practice Questions

1. Find the HCF of 24 and 36 using prime factorization.
2. Are 18 and 35 co-prime numbers? Give a reason.
3. Write the prime factorization of 72.
4. List all the prime numbers between 20 and 50.
5. Find the HCF of 48 and 60 using the factor method.

Answer Key

1. $24 = 2 \times 2 \times 2 \times 3$ and $36 = 2 \times 2 \times 3 \times 3$. Therefore, $\text{HCF} = 12$.
2. Yes. The only common factor of 18 and 35 is 1.
3. $72 = 2 \times 2 \times 2 \times 3 \times 3$.
4. Prime numbers between 20 and 50 are 23, 29, 31, 37, 41, 43, and 47.
5. The HCF of 48 and 60 is 12.