

Binomial Theorem

Introduction

The **Binomial Theorem** provides a simple method to expand expressions of the form $(a + b)^n$, where n is a positive integer. It helps in finding the expansion without repeated multiplication and is widely used in algebra, calculus, probability, and higher mathematics.

Binomial Expression

What is a Binomial?

A **binomial** is an algebraic expression containing **two terms**.

Examples:

- $x + y$
- $a - b$
- $2x + 3$

The Binomial Theorem is used to expand powers of such expressions.

Binomial Theorem

Statement of the Theorem

The Binomial Theorem states that:

$(a + b)^n$ can be expanded as a sum of terms involving **binomial coefficients**.

Each term contains:

- A binomial coefficient.
- A power of **a**.
- A power of **b**.

The exponent of **a** decreases while the exponent of **b** increases in each successive term.

Expansion of Binomial Expression

General Term

The expansion of $(a + b)^n$ contains $(n + 1)$ terms.

The general term is written as:

$$T_{r+1} = {}^nC_r a^{n-r} b^r$$

where:

- ${}^nC_r$ is the binomial coefficient.
- $r = 0, 1, 2, \dots, n$

Binomial Coefficients

Meaning

The coefficients in the expansion are called **binomial coefficients**.

They are given by:

$${}^nC_r = n! / [r!(n - r)!]$$

Important properties:

- ${}^nC_0 = 1$
- ${}^nC_n = 1$
- ${}^nC_r = {}^nC_{n-r}$

Pascal's Triangle

Finding Binomial Coefficients

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Pascal's Triangle is used to find the binomial coefficients quickly.

Each number in the triangle is obtained by adding the two numbers directly above it.

The coefficients in each row correspond to the coefficients of $(a + b)^n$.

Properties of Binomial Expansion

Important Facts

- Total number of terms = $n + 1$
- Sum of exponents in each term is always n .
- The exponent of the first term decreases successively.
- The exponent of the second term increases successively.
- The coefficients follow Pascal's Triangle.

Middle Term(s)

Finding the Middle Term

The number of middle terms depends on **n**.

- If **n** is even, there is **one middle term**.
- If **n** is odd, there are **two middle terms**.

Applications of Binomial Theorem

Importance

The Binomial Theorem is used in:

- Algebra
- Calculus
- Probability
- Statistics
- Engineering
- Scientific calculations

It helps simplify lengthy algebraic expansions.

Key Points

- A binomial has two terms.
- The Binomial Theorem expands **$(a + b)^n$** .
- The expansion contains **$(n + 1)$** terms.
- The general term is **$T_{r+1} = {}^nC_r a^{n-r} b^r$** .
- Binomial coefficients are obtained using **${}^nC_r$** .
- Pascal's Triangle helps determine the coefficients.
- The theorem has many applications in higher mathematics.

Chapter Summary

The Binomial Theorem provides a systematic method for expanding powers of binomial expressions. The chapter explains binomial coefficients, the general term, properties of the expansion, Pascal's Triangle, and the determination of middle terms. These concepts are useful in algebra, calculus, probability, and many advanced mathematical applications.