

Complex Numbers and Quadratic Equations

Introduction

The chapter **Complex Numbers and Quadratic Equations** extends the concept of real numbers by introducing **imaginary and complex numbers**. It also explains how quadratic equations can be solved even when they have no real roots. Complex numbers are widely used in mathematics, engineering, physics, and electrical sciences.

Complex Numbers

What is a Complex Number?

A **complex number** is a number of the form:

$$z = a + ib$$

where:

- **a** = Real Part (Re z)
- **b** = Imaginary Part (Im z)
- **i** = $\sqrt{-1}$, called the **imaginary unit**

Since $\sqrt{-1}$ is not a real number, $i^2 = -1$.

Types of Complex Numbers

Different Forms

Complex numbers are classified as:

- **Purely Real Number:** Imaginary part is zero. (Example: $5 + 0i$)
 - **Purely Imaginary Number:** Real part is zero. (Example: $4i$)
 - **General Complex Number:** Both real and imaginary parts are present. (Example: $3 + 2i$)
 - **Zero Complex Number:** Both real and imaginary parts are zero. ($0 + 0i$)
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Equality of Complex Numbers

Condition for Equality

Two complex numbers are equal if:

- Their real parts are equal.
- Their imaginary parts are equal.

If:

$$a + ib = c + id$$

then,

- $a = c$
- $b = d$

Algebra of Complex Numbers

Basic Operations

Operations on complex numbers include:

- **Addition**
- **Subtraction**
- **Multiplication**
- **Division**

While multiplying complex numbers, use the identity:

$$i^2 = -1$$

Conjugate of a Complex Number

Complex Conjugate

The **conjugate** of

$$z = a + ib$$

is

$$\bar{z} = a - ib$$

Important properties:

- Product of a complex number and its conjugate is always a **real number**.
- Conjugates are useful in simplifying fractions involving complex numbers.

Modulus of a Complex Number

Magnitude

The **modulus** of

$$z = a + ib$$

is given by:

$$|z| = \sqrt{a^2 + b^2}$$

The modulus represents the distance of the complex number from the origin on the Argand Plane.

Argand Plane

Geometrical Representation

A complex number is represented on the **Argand Plane**, where:

- Horizontal axis $\rightarrow$ Real Axis
- Vertical axis $\rightarrow$ Imaginary Axis

Each complex number corresponds to a unique point on the plane.

Quadratic Equations

Standard Form

A **quadratic equation** is written as:

$$ax^2 + bx + c = 0$$

where:

- $a \neq 0$
 - a, b, c are real numbers.
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Solution of Quadratic Equations

Quadratic Formula

The roots of

$$ax^2 + bx + c = 0$$

are obtained using:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The quantity $b^2 - 4ac$ is called the **Discriminant (D)**.

Nature of Roots

Discriminant

The discriminant determines the type of roots.

- $D > 0$: Two distinct real roots.
- $D = 0$: Two equal real roots.
- $D < 0$: Two complex (imaginary) roots.

Key Points

- A complex number is written as $a + ib$.
- $i^2 = -1$.
- Every complex number has a real part and an imaginary part.
- The conjugate changes only the sign of the imaginary part.
- The modulus gives the distance from the origin.
- The Argand Plane represents complex numbers graphically.
- A quadratic equation has the form $ax^2 + bx + c = 0$.
- The discriminant determines the nature of the roots.

Chapter Summary

This chapter introduces **complex numbers**, their algebra, conjugates, modulus, and graphical representation on the Argand Plane. It also explains quadratic equations and their solutions using the quadratic formula. The discriminant helps determine whether the roots are real or complex. These concepts are essential for higher mathematics, engineering, and physics.