

# Introduction to Three-Dimensional Geometry

## *Introduction*

**Three-Dimensional (3D) Geometry** is the branch of mathematics that studies the position of points, lines, and planes in **three-dimensional space**. Unlike two-dimensional geometry, 3D geometry uses **three coordinates** to locate a point. It is widely used in engineering, architecture, computer graphics, physics, navigation, and robotics.

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## Three-Dimensional Coordinate System

### *Cartesian Coordinate System*

A **3D Cartesian Coordinate System** consists of **three mutually perpendicular axes**:

- **X-axis**
- **Y-axis**
- **Z-axis**

These axes intersect at a common point called the **Origin (0, 0, 0)**.

Every point in space is represented by an **ordered triplet (x, y, z)**.

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## Coordinates of a Point

### *Position of a Point*

A point in three-dimensional space is represented as:

**P(x, y, z)**

where:

- **x** = Distance from the YZ-plane
- **y** = Distance from the XZ-plane
- **z** = Distance from the XY-plane

The three coordinates together determine the exact position of a point.

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## Octants

### *Division of Space*

The three coordinate planes divide the space into **eight regions**, called **Octants**.

Each octant has a different combination of positive and negative values of **x**, **y**, and **z** coordinates.

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## Coordinate Planes

### *Three Coordinate Planes*

The three coordinate planes are:

- **XY-plane**
- **YZ-plane**
- **XZ-plane**

These planes divide the three-dimensional space into different regions and help locate points.

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## Distance Between Two Points

### *Distance Formula*

If two points are:

**P(x<sub>1</sub>, y<sub>1</sub>, z<sub>1</sub>)** and **Q(x<sub>2</sub>, y<sub>2</sub>, z<sub>2</sub>)**

then the distance between them is:

$$PQ = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$$

This formula is an extension of the distance formula used in two-dimensional geometry.

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## Section Formula

### *Division of a Line Segment*

If a point divides the line segment joining two points in the ratio **m : n**, then its coordinates are found using the **Section Formula**.

It is useful for finding the coordinates of a point lying between two given points.

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## Midpoint Formula

### *Midpoint of a Line Segment*

The midpoint of the line joining:

**P(x<sub>1</sub>, y<sub>1</sub>, z<sub>1</sub>)** and **Q(x<sub>2</sub>, y<sub>2</sub>, z<sub>2</sub>)**

is:

$$[(x_1 + x_2)/2, (y_1 + y_2)/2, (z_1 + z_2)/2]$$

The midpoint divides the line segment into two equal parts.

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## Applications of 3D Geometry

### *Importance*

Three-dimensional geometry is used in:

- Engineering
- Architecture
- Computer Graphics
- Robotics
- Navigation
- Physics
- Satellite Communication
- Game Design

It helps represent and analyze objects in space.

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## Key Points

- Three-dimensional geometry uses **x, y, and z** coordinates.
  - The three axes intersect at the **origin (0, 0, 0)**.
  - Space is divided into **eight octants**.
  - Three coordinate planes are **XY, YZ, and XZ**.
  - Distance between two points is calculated using the 3D distance formula.
  - The midpoint formula finds the middle point of a line segment.
  - 3D geometry has wide applications in science and technology.
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## Chapter Summary

This chapter introduces the three-dimensional coordinate system and explains how points are represented using **x, y, and z coordinates**. It covers coordinate planes, octants, the distance formula, section formula, and midpoint formula. Three-dimensional geometry is an important foundation for higher mathematics and has numerous applications in engineering, architecture, computer graphics, robotics, and modern technology.