Limits and Derivatives

Introduction

Limits and Derivatives form the foundation of **Calculus**. A **limit** describes the value that a function approaches as the input approaches a particular value, while a **derivative** measures the rate at which a function changes. These concepts are widely used in mathematics, physics, engineering, economics, and many other fields.

Limit

What is a Limit?

A **limit** is the value that a function approaches as the variable approaches a particular point.

It is written as:

lim (x → a) f(x) = L

This means that as **x** approaches **a**, the value of **f(x)** approaches **L**.

Limits help study the behavior of functions near a point.

Properties of Limits

Basic Properties

The important properties of limits are:

- Limit of a constant is the constant itself.
- Limit of a sum is the sum of the limits.
- Limit of a difference is the difference of the limits.
- Limit of a product is the product of the limits.
- Limit of a quotient is the quotient of the limits, provided the denominator is not zero.

These properties simplify the evaluation of limits.

Standard Limits

Important Formulae

Some standard limits are commonly used in calculations.

Examples include:

- **$\lim_{x \to 0} (\sin x / x) = 1$**
- **$\lim_{x \to 0} ((1 - \cos x) / x^2) = 1/2$**
- **$\lim_{x \to 0} ((e^x - 1) / x) = 1$**

These formulas are useful in solving problems involving trigonometric and exponential functions.

Derivative

Definition

The **derivative** of a function represents its **instantaneous rate of change** or the **slope of the tangent** at a point on the curve.

It is denoted by:

- **dy/dx**
- **$f'(x)$**

Derivative from First Principles

Definition Using Limit

The derivative of a function is defined as:

$f'(x) = \lim_{h \to 0} [f(x + h) - f(x)] / h$

This is known as the **First Principle of Differentiation**.

Basic Rules of Differentiation

Rules

The commonly used differentiation rules are:

- Constant Rule
- Power Rule
- Sum Rule
- Difference Rule
- Product Rule
- Quotient Rule
- Chain Rule

These rules help differentiate different types of functions easily.

Derivatives of Standard Functions

Common Derivatives

Some standard derivatives are:

- **$d/dx\ (x^n) = nx^{n-1}$**
- **$d/dx\ (\sin x) = \cos x$**
- **$d/dx\ (\cos x) = -\sin x$**
- **$d/dx\ (e^x) = e^x$**
- **$d/dx\ (\ln x) = 1/x$**

These formulas are frequently used in solving calculus problems.

Applications of Derivatives

Importance

Derivatives are used in:

- Finding the slope of a curve.
- Calculating velocity and acceleration.
- Optimization problems.
- Economics.
- Engineering.
- Physics.

They help analyze how quantities change with time or position.

Key Points

- Limits describe the behavior of a function near a point.
- Derivatives measure the rate of change of a function.
- The derivative is defined using limits.
- Standard limits simplify calculations.
- Differentiation follows several important rules.
- Derivatives are widely used in science, engineering, and economics.
- Calculus begins with the concepts of limits and derivatives.

Chapter Summary

This chapter introduces the concepts of limits and derivatives, which form the basis of calculus. It explains the definition and properties of limits, standard limit formulas, and the derivative as the instantaneous rate of change of a function. The chapter also covers differentiation from first principles, basic differentiation rules, derivatives of standard functions, and practical applications in mathematics, physics, engineering, and economics.