

Mathematical Reasoning

Introduction

Mathematical Reasoning is the process of using logical thinking to arrive at correct conclusions. It helps us analyze statements, identify patterns, and solve problems systematically. This chapter introduces different types of statements, logical connectives, implications, and methods of reasoning used in mathematics.

Statement

What is a Statement?

A **statement** is a sentence that is either **true** or **false**, but not both.

Examples:

- "7 is a prime number." (True)
- "5 is an even number." (False)

Questions, commands, and exclamatory sentences are **not** statements because they cannot be assigned a truth value.

Logical Connectives

Connecting Statements

Two or more statements can be combined using logical connectives.

The main logical connectives are:

- **AND** ($\wedge$) – Conjunction
- **OR** ($\vee$) – Disjunction
- **NOT** ($\neg$) – Negation

These connectives are used to form compound statements.

Truth Table

Truth Values

A **truth table** shows the truth value of a compound statement for all possible truth values of its components.

Truth tables help determine whether a statement is always true, always false, or depends on the given conditions.

Implication

If-Then Statement

An **implication** is a statement of the form:

If p, then q ($p \rightarrow q$)

Where:

- **p** is called the **hypothesis**.
- **q** is called the **conclusion**.

The implication is false only when **p is true** and **q is false**.

Converse, Inverse and Contrapositive

Related Statements

For the statement $p \rightarrow q$:

- **Converse:** $q \rightarrow p$
- **Inverse:** $\neg p \rightarrow \neg q$
- **Contrapositive:** $\neg q \rightarrow \neg p$

The **contrapositive** always has the same truth value as the original implication.

Logical Equivalence

Equivalent Statements

Two statements are **logically equivalent** if they have the same truth values in every possible case.

Logical equivalence is useful for simplifying mathematical expressions and proofs.

Methods of Reasoning

Deductive and Inductive Reasoning

Deductive Reasoning:

- Moves from general statements to specific conclusions.

- Produces logically certain conclusions if the premises are true.

Inductive Reasoning:

- Moves from specific observations to general conclusions.
 - Conclusions are based on patterns and may not always be true.
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Validating Statements

Checking Arguments

Mathematical reasoning is used to test whether an argument is valid.

An argument is considered **valid** if the conclusion logically follows from the given premises.

Applications of Mathematical Reasoning

Importance

Mathematical reasoning is used in:

- Solving mathematical problems.
 - Computer programming.
 - Artificial Intelligence.
 - Scientific research.
 - Logical decision-making.
 - Everyday problem-solving.
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Key Points

- A statement is either true or false.
 - Logical connectives combine simple statements.
 - Truth tables determine the truth value of compound statements.
 - Implication is written as $p \rightarrow q$.
 - Converse, inverse, and contrapositive are related statements.
 - Deductive and inductive reasoning are important methods of logical thinking.
 - Mathematical reasoning develops analytical and problem-solving skills.
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Chapter Summary

Mathematical Reasoning introduces logical thinking through statements, truth values, logical connectives, implications, and methods of reasoning. It helps students analyze arguments, verify conclusions, and solve problems systematically. These concepts form the foundation of higher mathematics, computer science, and logical decision-making.