Motion in a Plane

Introduction

Motion in a plane refers to the motion of an object in two dimensions, such as the motion of a projectile or a car moving on a curved road. To study two-dimensional motion, we use **vectors, position and displacement, velocity and acceleration, projectile motion, and uniform circular motion**.

Scalars and Vectors

Scalar Quantity

A **scalar quantity** has only magnitude and no direction.

Examples:

- Distance
- Speed
- Mass
- Time
- Temperature

Vector Quantity

A **vector quantity** has both magnitude and direction.

Examples:

- Displacement
- Velocity
- Acceleration
- Force
- Momentum

A vector is represented by an arrow. The length represents its magnitude and the arrowhead represents its direction.

Representation of a Vector

Unit Vector

A **unit vector** is a vector having magnitude one. It is used to represent the direction of a vector.

Along the x, y and z axes, unit vectors are represented by:

$\mathbf{\hat{i}}, \mathbf{\hat{j}}, \mathbf{\hat{k}}$

A vector **A** can be written as:

$$\mathbf{A} = A_x\hat{i} + A_y\hat{j} + Az\hat{k}$$

Addition of Vectors

Triangle Law

According to the **triangle law of vector addition**, if two vectors are represented by two sides of a triangle taken in order, their resultant is represented by the third side taken in the opposite direction.

Parallelogram Law

If two vectors acting at a point are represented by the adjacent sides of a parallelogram, their resultant is represented by the diagonal of the parallelogram.

For two vectors **A** and **B** making an angle **θ**:

$$\mathbf{R} = \sqrt{(A^2 + B^2 + 2AB\cos\theta)}$$

Resolution of a Vector

Definition

The process of splitting a vector into its components along two mutually perpendicular directions is called **resolution of a vector**.

For a vector **A** making an angle **θ** with the x-axis:

$$A_x = A\cos\theta$$

$$A_y = A\sin\theta$$

Therefore:

$$\mathbf{A} = A_x\hat{i} + A_y\hat{j}$$

The magnitude is:

$$A = \sqrt{(A_x^2 + A_y^2)}$$

Position and Displacement

Position Vector

The position of a particle with respect to the origin is represented by its **position vector**.

$$\mathbf{r} = x\hat{i} + y\hat{j}$$

Displacement

Displacement is the change in the position of a particle.

$$\Delta\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1$$

Displacement is a vector quantity.

Velocity and Acceleration

Velocity

The rate of change of displacement with time is called **velocity**.

$$\mathbf{v} = d\mathbf{r}/dt$$

Average velocity:

$$\mathbf{v_{avg}} = \Delta\mathbf{r}/\Delta t$$

Acceleration

The rate of change of velocity with time is called **acceleration**.

$$\mathbf{a} = d\mathbf{v}/dt$$

Average acceleration:

$$\mathbf{a_{avg}} = \Delta\mathbf{v}/\Delta t$$

In two-dimensional motion:

$$\mathbf{v} = v_x\hat{i} + v_y\hat{j}$$

$$\mathbf{a} = a_x\hat{i} + a_y\hat{j}$$

Projectile Motion

Definition

The motion of an object projected into the air under the influence of gravity alone is called **projectile motion**.

Examples:

- A ball thrown at an angle
- A stone thrown from a height
- A football kicked into the air

Projectile motion is a combination of:

- Horizontal motion with constant velocity
- Vertical motion with constant acceleration due to gravity

Projectile Motion Equations

If a projectile is launched with initial velocity **u** at an angle **θ**:

Horizontal component:

$$u_x = u \cos\theta$$

Vertical component:

$$u_y = u \sin\theta$$

Time of Flight

The total time for which the projectile remains in the air is:

$$T = 2u \sin\theta/g$$

Maximum Height

The maximum vertical height reached is:

$$H = u^2\sin^2\theta/2g$$

Horizontal Range

The horizontal distance travelled is:

$$R = u^2\sin 2\theta/g$$

The range is maximum when:

$\boldsymbol{\theta = 45^\circ}$

Uniform Circular Motion

Definition

When an object moves along a circular path with constant speed, its motion is called **uniform circular motion**.

Although speed remains constant, velocity changes continuously because its direction changes.

Therefore, the object has acceleration.

Centripetal Acceleration

The acceleration directed towards the centre of the circular path is called **centripetal acceleration**.

$\mathbf{ac = v^2/r}$

It can also be written as:

$\mathbf{ac = \omega^2 r}$

where:

- **v** = Linear speed
- **r** = Radius
- **ω** = Angular velocity

Angular Velocity

Definition

The rate of change of angular displacement with time is called **angular velocity**.

$\boldsymbol{\omega = d\theta/dt}$

Relation between linear and angular velocity:

$\mathbf{v = r\omega}$

The direction of angular velocity is perpendicular to the plane of rotation.

Centripetal Force

Definition

The force required to keep an object moving in a circular path is called **centripetal force**.

$$F_c = mv^2/r$$

It is always directed towards the centre of the circular path.

Examples:

- Tension in a string when a stone is rotated
- Gravitational force keeping planets in orbit
- Friction providing the centripetal force for a vehicle on a curved road

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Key Points

- Motion in a plane is two-dimensional motion.
- Scalars have magnitude only, while vectors have magnitude and direction.
- Vectors can be added using the triangle or parallelogram law.
- A vector can be resolved into perpendicular components.
- Projectile motion has independent horizontal and vertical components.
- The maximum range of a projectile occurs at **45°**.
- In uniform circular motion, speed is constant but velocity changes.
- Centripetal acceleration is directed towards the centre.
- $a_c = v^2/r$
- $v = r\omega$
- Centripetal force is $F_c = mv^2/r$.

Chapter Summary

This chapter introduces the concepts required to study two-dimensional motion. It explains scalars and vectors, vector addition and resolution, position and displacement vectors, velocity and acceleration, projectile motion, and uniform circular motion. The study of motion in a plane is essential for understanding the motion of projectiles, vehicles, satellites, planets, and other objects moving in two dimensions.