Oscillations

Introduction

Oscillation is the repeated to-and-fro motion of an object about a fixed or mean position. Oscillations are seen in many systems, such as a simple pendulum, a vibrating string, and a mass attached to a spring. This chapter explains **periodic motion, simple harmonic motion, time period, frequency, amplitude, phase, energy in SHM, and simple pendulum**.

Periodic Motion

Definition

A motion that repeats itself after equal intervals of time is called **periodic motion**.

Examples:

- Motion of Earth around the Sun
- Motion of a simple pendulum
- Vibration of a tuning fork
- Motion of a mass attached to a spring

The time taken to complete one cycle is called the **time period (T)**.

Oscillatory Motion

Definition

The repeated to-and-fro motion of an object about its mean position is called **oscillatory motion**.

An oscillating body moves between two extreme positions through its mean position.

Important Terms

- **Mean Position:** The equilibrium position of the body.
- **Extreme Position:** The farthest position from the mean position.
- **Amplitude (A):** Maximum displacement from the mean position.
- **Time Period (T):** Time taken for one complete oscillation.
- **Frequency (f):** Number of oscillations completed per second.

Relation:

$f = 1/T$

SI Unit of frequency: **Hertz (Hz)**

Simple Harmonic Motion

Definition

Simple Harmonic Motion (SHM) is a type of oscillatory motion in which the restoring force or acceleration is directly proportional to the displacement from the mean position and is always directed towards the mean position.

Mathematically:

$$F \propto -x$$

or

$$F = -kx$$

The negative sign indicates that the restoring force acts opposite to the displacement.

Characteristics of SHM

Main Features

- Motion is periodic.
- The body oscillates about a mean position.
- Restoring force acts towards the mean position.
- Restoring force is proportional to displacement.
- The acceleration is maximum at extreme positions.
- The velocity is maximum at the mean position.
- The velocity is zero at the extreme positions.

Displacement, Velocity and Acceleration

Displacement

The displacement of a particle performing SHM can be represented as:

$$x = A \sin(\omega t + \varphi)$$

where:

- **A** = Amplitude
- **ω** = Angular frequency
- **t** = Time
- **φ** = Initial phase

Angular Frequency

$\boldsymbol{\omega = 2\pi f = 2\pi/T}$

The SI unit of angular frequency is **rad/s**.

Acceleration

In SHM:

$\boldsymbol{a = -\omega^2 x}$

This shows that acceleration is directly proportional to displacement and is directed towards the mean position.

Energy in SHM

Kinetic Energy

The kinetic energy of an oscillating body is:

K.E. = $\boldsymbol{\frac{1}{2}mv^2}$

It is maximum at the mean position and zero at the extreme positions.

Potential Energy

Potential energy is maximum at the extreme positions and minimum at the mean position.

For a spring:

P.E. = $\boldsymbol{\frac{1}{2}kx^2}$

Total Energy

The total mechanical energy remains constant in ideal SHM.

Total Energy = K.E. + P.E.

For a spring-mass system:

$\boldsymbol{E = \frac{1}{2}kA^2}$

Spring-Mass System

Time Period

For a body of mass **m** attached to a spring of force constant **k**:

$$T = 2\pi\sqrt{(m/k)}$$

The time period increases with mass and decreases with the stiffness of the spring.

Simple Pendulum

Definition

A **simple pendulum** consists of a small bob suspended from a fixed point by a light, inextensible string.

For small oscillations, the motion of a simple pendulum is approximately SHM.

Time Period

$$T = 2\pi\sqrt{(l/g)}$$

where:

- **l** = Length of pendulum
- **g** = Acceleration due to gravity

The time period:

- Increases with the length of the pendulum.
- Decreases when the value of **g** increases.
- Does not depend on the mass of the bob.
- Is independent of amplitude for small oscillations.

Damped Oscillations

Definition

When the amplitude of oscillation gradually decreases due to resistive forces such as friction or air resistance, the oscillations are called **damped oscillations**.

The energy of the system gradually decreases.

Example: A pendulum eventually stops due to air resistance and friction.

Forced Oscillations

Definition

When an external periodic force continuously acts on an oscillating system, the resulting oscillations are called **forced oscillations**.

The system oscillates with the frequency of the external force.

Resonance

Definition

Resonance occurs when the frequency of the external periodic force becomes equal or nearly equal to the natural frequency of the system.

At resonance, the amplitude of oscillation becomes very large.

Applications

Resonance is used in:

- Musical instruments
- Radio tuning
- Electrical circuits
- Vibrating systems

Resonance can also be harmful because excessive vibrations may damage structures.

Applications of Oscillations

Importance

Oscillations are used in:

- Clocks and watches
- Musical instruments
- Seismometers
- Radio and communication systems
- Electrical circuits
- Engineering and machinery

Key Points

- Oscillation is repeated to-and-fro motion about a mean position.
- Periodic motion repeats after equal time intervals.

- SHM has a restoring force proportional to displacement.
- The restoring force always acts towards the mean position.
- **$f = 1/T$**
- **$\omega = 2\pi f$**
- Velocity is maximum at the mean position.
- Acceleration is maximum at the extreme positions.
- Total energy in ideal SHM remains constant.
- Time period of a spring system is **$T = 2\pi\sqrt{(m/k)}$**.
- Time period of a simple pendulum is **$T = 2\pi\sqrt{(l/g)}$**.
- Resonance produces large amplitude when driving frequency matches natural frequency.

Chapter Summary

This chapter explains oscillatory and periodic motion and introduces the concept of Simple Harmonic Motion (SHM). It covers amplitude, time period, frequency, angular frequency, displacement, velocity, acceleration, energy in SHM, spring-mass systems, simple pendulums, damped and forced oscillations, and resonance. Oscillations are important in many natural phenomena, mechanical systems, electrical circuits, clocks, and communication technologies.