

Principle of Mathematical Induction

Introduction

The **Principle of Mathematical Induction (PMI)** is a mathematical technique used to prove statements that are true for all natural numbers. Instead of checking each value individually, mathematical induction proves that if a statement is true for the first natural number and remains true for the next number, then it is true for all natural numbers.

Mathematical Induction

What is Mathematical Induction?

Mathematical induction is a method of proof used to verify statements involving **natural numbers**.

It is based on the idea that if the first step is correct and each step leads to the next, then the entire sequence of steps is correct.

Principle of Mathematical Induction

Statement of the Principle

Let $P(n)$ be a statement involving a natural number n .

To prove that $P(n)$ is true for all natural numbers, two steps are followed:

1. **Base Case:** Show that $P(1)$ (or the first given value) is true.
2. **Induction Step:** Assume $P(k)$ is true for some natural number k , then prove that $P(k + 1)$ is also true.

If both steps are satisfied, then $P(n)$ is true for all natural numbers.

Steps of Mathematical Induction

Procedure

The process of mathematical induction consists of:

- Writing the given statement $P(n)$.
 - Verifying the **base case**.
 - Assuming $P(k)$ is true (Induction Hypothesis).
 - Proving $P(k + 1)$ using the assumption.
 - Concluding that the statement is true for all natural numbers.
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Induction Hypothesis

Assumption in the Proof

The assumption that $P(k)$ is true for some natural number k is called the **Induction Hypothesis**.

This assumption is used only to prove the next case, $P(k + 1)$.

Applications of Mathematical Induction

Where is it Used?

Mathematical induction is commonly used to prove:

- Algebraic identities.
 - Formulae involving sums.
 - Divisibility properties.
 - Inequalities.
 - Statements involving natural numbers.
 - Recursive relations.
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Advantages of Mathematical Induction

Importance

Mathematical induction:

- Proves infinitely many cases using only two steps.
 - Saves time and effort.
 - Provides a logical and systematic method of proof.
 - Is widely used in higher mathematics and computer science.
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Important Points

Remember

- Always verify the base case first.
 - The induction hypothesis should not be treated as the final proof.
 - The proof is complete only after proving $P(k + 1)$.
 - Mathematical induction is applicable only to statements involving natural numbers.
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Key Points

- Mathematical induction proves statements for all natural numbers.

- The proof consists of the **base case** and the **induction step**.
 - The assumption **$P(k)$** is called the induction hypothesis.
 - If both steps are satisfied, the statement is true for every natural number.
 - PMI is widely used in algebra, number theory, and higher mathematics.
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Chapter Summary

The Principle of Mathematical Induction is an important proof technique used to establish the truth of mathematical statements involving natural numbers. The method involves verifying the first case and then proving that if the statement is true for one natural number, it is also true for the next. This principle provides a simple and powerful way to prove results that hold for infinitely many natural numbers.