

Relations and Functions

Introduction

Relations and Functions are fundamental concepts in mathematics that describe the relationship between two sets. They help us understand how elements of one set are connected to elements of another set. These concepts are widely used in algebra, calculus, computer science, and real-life applications.

Cartesian Product

Product of Two Sets

The **Cartesian Product** of two non-empty sets **A** and **B** is the set of all ordered pairs **(a, b)** where **a** $\in$ **A** and **b** $\in$ **B**.

It is denoted by:

$$\mathbf{A \times B = \{(a, b) : a \in A \text{ and } b \in B\}}$$

If **n(A) = m** and **n(B) = n**, then:

$$\mathbf{n(A \times B) = m \times n}$$

Relation

What is a Relation?

A **relation** from set **A** to set **B** is any subset of the Cartesian product **A** $\times$ **B**.

A relation shows how the elements of one set are connected to the elements of another set.

Types of Relations

Important Relations

Some common types of relations are:

- **Empty Relation:** Contains no ordered pairs.
 - **Universal Relation:** Contains all ordered pairs of **A** $\times$ **A**.
 - **Identity Relation:** Every element is related to itself.
 - **Inverse Relation:** Obtained by reversing the ordered pairs.
-

Function

What is a Function?

A **function** is a special type of relation in which **every element of the domain is associated with exactly one element of the codomain**.

It is written as:

$$f : A \rightarrow B$$

where:

- **A** = Domain
 - **B** = Codomain
 - **f(A)** = Range
-

Domain, Codomain and Range

Basic Terms

- **Domain:** Set of all input values.
- **Codomain:** Set of possible output values.
- **Range:** Set of actual output values.

The **Range** is always a subset of the **Codomain**.

Types of Functions

Different Functions

Important types of functions include:

- **One-One (Injective) Function:** Different inputs have different outputs.
 - **Many-One Function:** Two or more inputs have the same output.
 - **Onto (Surjective) Function:** Every element of the codomain has at least one pre-image.
 - **Into Function:** Some elements of the codomain have no pre-image.
 - **One-One Onto (Bijective) Function:** Both one-one and onto.
-

Graphs of Functions

Graphical Representation

Functions can be represented graphically on the Cartesian plane.

The graph helps us understand:

- Domain
- Range
- Behaviour of the function

A graph represents a function only if it passes the **Vertical Line Test**.

Algebra of Real Functions

Operations on Functions

If $f(x)$ and $g(x)$ are two functions, then:

- $(f + g)(x) = f(x) + g(x)$
- $(f - g)(x) = f(x) - g(x)$
- $(fg)(x) = f(x) \times g(x)$
- $(f/g)(x) = f(x) \div g(x)$ (where $g(x) \neq 0$)

These operations help in solving mathematical problems involving functions.

Applications of Functions

Importance

Functions are used in:

- Mathematics
- Physics
- Economics
- Engineering
- Computer Science
- Statistics

They help represent relationships between different quantities.

Key Points

- The Cartesian product consists of ordered pairs.
 - A relation is a subset of the Cartesian product.
 - Every function is a relation, but every relation is not a function.
 - A function assigns exactly one output to each input.
 - Domain, codomain, and range are important concepts.
 - Functions are classified as one-one, many-one, onto, into, and bijective.
 - Functions are widely used in mathematics and science.
-

Chapter Summary

The chapter introduces Cartesian products, relations, and functions. A relation is a connection between two sets, while a function is a special relation where each input has exactly one output. The chapter also explains different types of relations and functions, domain, codomain, range, and operations on functions. These concepts form the foundation for advanced topics in mathematics.