Sequences and Series

Introduction

A **Sequence** is an ordered list of numbers that follow a specific rule, while a **Series** is the sum of the terms of a sequence. Sequences and series are widely used in mathematics, physics, finance, engineering, and computer science to study patterns and solve practical problems.

Sequence

What is a Sequence?

A **sequence** is an ordered arrangement of numbers in which each number is called a **term**.

Examples:

- 2, 4, 6, 8, ...
- 1, 3, 9, 27, ...

The terms of a sequence are usually represented by $\mathbf{a_1, a_2, a_3, ..., a_n}$.

Series

What is a Series?

A **series** is obtained by adding the terms of a sequence.

Examples:

- 2 + 4 + 6 + 8 + ...
- 1 + 3 + 9 + 27 + ...

The sum of a finite series is represented by $\mathbf{S_n}$.

Arithmetic Progression (AP)

Definition

An **Arithmetic Progression (AP)** is a sequence in which the difference between consecutive terms is constant.

This constant difference is called the **Common Difference (d)**.

General form:

a, a + d, a + 2d, a + 3d, ...

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nth Term of AP

Formula

The **nth term** of an Arithmetic Progression is:

$$a_n = a + (n - 1)d$$

where:

- **a** = First term
- **d** = Common difference
- **n** = Number of terms

Sum of n Terms of AP

Formula

The sum of the first **n** terms of an AP is:

$$S_n = n/2\ [2a + (n - 1)d]$$

or

$$S_n = n/2\ (a + l)$$

where **l** is the last term.

Geometric Progression (GP)

Definition

A **Geometric Progression (GP)** is a sequence in which each term is obtained by multiplying the previous term by a constant.

This constant is called the **Common Ratio (r)**.

General form:

a, ar, ar^2, ar^3, ...

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nth Term of GP

Formula

The **nth term** of a GP is:

$$a_n = ar^{n-1}$$

where:

- **a** = First term
- **r** = Common ratio

Sum of n Terms of GP

Formula

If $r \neq 1$, then:

$$S_n = a(r^n - 1)/(r - 1)$$

or

$$S_n = a(1 - r^n)/(1 - r)$$

Infinite Geometric Series

Sum to Infinity

If $|r| < 1$, the sum of an infinite GP is:

$$S_\infty = a/(1 - r)$$

This formula is valid only when the common ratio lies between **−1 and 1**.

Applications of Sequences and Series

Importance

Sequences and series are used in:

- Mathematics
- Finance and Banking
- Engineering
- Physics

- Computer Science
- Population Growth
- Compound Interest

Key Points

- A sequence is an ordered list of numbers.
- A series is the sum of sequence terms.
- AP has a constant common difference.
- GP has a constant common ratio.
- Formula for nth term of AP: $\mathbf{a + (n - 1)d}$.
- Formula for nth term of GP: $\mathbf{ar^{n-1}}$.
- Infinite GP exists only when $\mathbf{|r| < 1}$.
- Sequences and series have many practical applications.

Chapter Summary

This chapter introduces sequences and series, Arithmetic Progression (AP), and Geometric Progression (GP). It explains the formulas for finding the nth term and the sum of terms in AP and GP. The concept of an infinite geometric series is also discussed. These topics are widely used in mathematics, finance, engineering, and scientific calculations.