System of Particles and Rotational Motion

Introduction

A **system of particles** is a collection of two or more particles considered together. The motion of such a system can be studied using the **centre of mass**. **Rotational motion** deals with the motion of a rigid body about a fixed axis. Important concepts include **centre of mass, torque, angular momentum, moment of inertia, and rotational equilibrium**.

Centre of Mass

Definition

The **centre of mass** of a system is the point where the entire mass of the system may be considered to be concentrated for studying its translational motion.

For two particles:

xcm = $(m_1x_1 + m_2x_2) / (m_1 + m_2)$

The centre of mass depends on the distribution of mass in the system.

Motion of Centre of Mass

Importance

The centre of mass of a system moves as if the **entire mass of the system** were concentrated at that point and all external forces acted on it.

The acceleration of the centre of mass is related to the net external force:

Fext = M acm

where **M** is the total mass and **acm** is the acceleration of the centre of mass.

Internal forces do not affect the motion of the centre of mass.

Linear Momentum

Definition

The **linear momentum** of a particle is the product of its mass and velocity.

p = mv

For a system, total momentum is the vector sum of the momenta of all particles.

If the net external force on a system is zero, its total linear momentum remains **conserved**.

Torque

Turning Effect of Force

Torque is the turning effect produced by a force about an axis or point.

$\tau = rF \sin \theta$

where:

- **r** = Distance from the axis
- **F** = Applied force
- **θ** = Angle between r and F

Torque is maximum when the force acts perpendicular to the position vector.

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Angular Momentum

Definition

Angular momentum is the rotational analogue of linear momentum.

For a particle:

$L = r \times p$

For a rigid body rotating about a fixed axis:

$L = I\omega$

where:

- **I** = Moment of inertia
- **ω** = Angular velocity

If the external torque is zero, angular momentum remains conserved.

Moment of Inertia

Rotational Inertia

The **moment of inertia (I)** of a body measures its resistance to change in rotational motion.

For a system of particles:

$$\mathbf{I = \Sigma m_i r_i^2}$$

The moment of inertia depends on:

- Mass of the body
- Distribution of mass
- Position of the axis of rotation

Greater mass farther from the axis means greater moment of inertia.

Radius of Gyration

Definition

The **radius of gyration (K)** is the distance from the axis at which the entire mass of a body can be assumed to be concentrated without changing its moment of inertia.

$$\mathbf{I = MK^2}$$

where **M** is the total mass.

Rotational Motion

Angular Variables

Rotational motion is described using:

- **Angular Displacement (θ)**
- **Angular Velocity (ω)**
- **Angular Acceleration (α)**

The relationship between linear and angular quantities is:

$$\mathbf{v = r\omega}$$

and

$$\mathbf{a_t = r\alpha}$$

where **r** is the distance from the axis.

Rotational Equilibrium

Condition

A rigid body is in **rotational equilibrium** when the net torque acting on it is zero.

$$\Sigma\tau = 0$$

For complete equilibrium, both conditions must be satisfied:

- Net force = 0
- Net torque = 0

This principle is used in balancing structures and machines.

Moment of Inertia of Common Bodies

Important Results

Some common moments of inertia are:

- **Ring about its central axis:** $I = MR^2$
- **Disc about its central axis:** $I = \frac{1}{2}MR^2$
- **Solid sphere about its diameter:** $I = \frac{2}{5}MR^2$
- **Rod about its centre:** $I = \frac{1}{12}ML^2$
- **Rod about one end:** $I = \frac{1}{3}ML^2$

These values depend on the shape and axis of rotation.

Parallel and Perpendicular Axis Theorems

Parallel Axis Theorem

The moment of inertia about any axis parallel to an axis through the centre of mass is:

$$I = I_{cm} + Md^2$$

Perpendicular Axis Theorem

For a plane lamina:

$$I_z = I_x + I_y$$

These theorems help calculate the moment of inertia about different axes.

Rolling Motion

Definition

When a body rotates while moving forward without slipping, it undergoes **rolling motion**.

For pure rolling:

$v = R\omega$

The total kinetic energy of a rolling body is the sum of:

- Translational kinetic energy
- Rotational kinetic energy

Applications

Importance

The concepts of rotational motion are used in:

- Wheels and gears
- Engines
- Flywheels
- Rotating machines
- Balance and stability
- Satellites and planets
- Engineering structures

Key Points

- The centre of mass represents the average position of mass in a system.
- The motion of the centre of mass depends only on external forces.
- Linear momentum is conserved when net external force is zero.
- Torque produces rotational motion.
- Moment of inertia measures resistance to rotational motion.
- Angular momentum is conserved when external torque is zero.
- A body is in rotational equilibrium when net torque is zero.
- Pure rolling satisfies **$v = R\omega$**.

Chapter Summary

This chapter explains the motion of a system of particles and the rotational motion of rigid bodies. It introduces the centre of mass, linear momentum, torque, angular momentum, moment of inertia, radius of gyration, rotational equilibrium, and rolling motion. The conservation laws of linear momentum and angular momentum are important principles used to understand the motion of particles and rotating bodies.