

Trigonometric Functions

Introduction

Trigonometry is a branch of mathematics that studies the relationship between the angles and sides of triangles. In Class 11, trigonometric functions are extended from acute angles to angles of any measure using the unit circle. These functions are widely used in mathematics, physics, engineering, navigation, and astronomy.

Angle Measurement

Degree and Radian Measure

An angle can be measured in:

- Degree ($^{\circ}$)
- Radian (rad)

Important conversions:

- $180^{\circ} = \pi$ radians
- $360^{\circ} = 2\pi$ radians
- $1 \text{ radian} = 180^{\circ}/\pi$
- $1^{\circ} = \pi/180$ radians

Radians are commonly used in higher mathematics.

Trigonometric Functions

Six Trigonometric Functions

The six trigonometric functions are:

- Sine ($\sin \theta$)
- Cosine ($\cos \theta$)
- Tangent ($\tan \theta$)
- Cosecant ($\operatorname{cosec} \theta$)
- Secant ($\sec \theta$)
- Cotangent ($\cot \theta$)

For a right-angled triangle:

- $\sin \theta = \text{Perpendicular} / \text{Hypotenuse}$
- $\cos \theta = \text{Base} / \text{Hypotenuse}$
- $\tan \theta = \text{Perpendicular} / \text{Base}$
- $\operatorname{cosec} \theta = 1 / \sin \theta$
- $\sec \theta = 1 / \cos \theta$
- $\cot \theta = 1 / \tan \theta$

Trigonometric Functions of Any Angle

Using the Unit Circle

For angles greater than 90° or negative angles, trigonometric functions are defined using the **unit circle**.

Coordinates of a point on the unit circle:

- $x = \cos \theta$
- $y = \sin \theta$

This definition allows trigonometric functions for all real numbers.

Signs of Trigonometric Functions

Quadrants

The signs of trigonometric functions depend on the quadrant.

- **First Quadrant:** All functions are positive.
- **Second Quadrant:** Only sin and cosec are positive.
- **Third Quadrant:** Only tan and cot are positive.
- **Fourth Quadrant:** Only cos and sec are positive.

The mnemonic "**All Students Take Coffee**" (**ASTC**) helps remember the signs.

Important Identities

Basic Trigonometric Identities

Some important identities are:

- $\sin^2 \theta + \cos^2 \theta = 1$
- $\sec^2 \theta = 1 + \tan^2 \theta$
- $\operatorname{cosec}^2 \theta = 1 + \cot^2 \theta$
- $\tan \theta = \sin \theta / \cos \theta$
- $\cot \theta = \cos \theta / \sin \theta$

These identities are used to simplify expressions and solve equations.

Trigonometric Functions of Special Angles

Standard Values

Important angles include:

- 0°
- 30°
- 45°
- 60°
- 90°

Students should memorize the values of **sin**, **cos**, **tan**, **cosec**, **sec**, and **cot** for these angles.

Trigonometric Graphs

Graphs of Functions

The graphs of **sin x**, **cos x**, and **tan x** show how these functions change with the angle.

- **sin x** and **cos x** are periodic with period 2π .
- **tan x** is periodic with period π .

These graphs help understand the behaviour of trigonometric functions.

Applications of Trigonometry

Uses in Daily Life

Trigonometry is used in:

- Measuring heights and distances.
 - Engineering and construction.
 - Navigation.
 - Astronomy.
 - Surveying.
 - Computer graphics and physics.
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Key Points

- Angles are measured in degrees and radians.
- There are six trigonometric functions.
- The unit circle defines trigonometric functions for all angles.
- The sign of a function depends on the quadrant.
- Trigonometric identities help simplify expressions.
- Standard angle values are important for problem-solving.
- Trigonometric functions are periodic and have many practical applications.

Chapter Summary

Trigonometric functions describe the relationship between angles and sides of triangles and are extended to all angles using the unit circle. The chapter covers angle measurement, six trigonometric functions, signs in different quadrants, important identities, standard angle values, graphs, and practical applications. A strong understanding of these concepts forms the foundation for advanced mathematics, calculus, and engineering.